



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

if  $x = 1/e = 0.368$  approx. Therefore, the second differences in a table of values of this function must be practically zero for values of  $x$  near 0.368.<sup>1</sup>

Also solved by H. L. OLSON, H. S. UHLER, A. H. WILSON, and the Proposer.

**2842 [1920, 274]. Proposed by H. S. UHLER, Yale University.**

Express explicitly the following sextic in  $x$  as the product of a quadratic and a biquadratic:

$$3x^6 - 6k_1x^5 + (7k_1^2 - 9k_2^2)x^4 - 2(2k_1^3 - 4k_1k_2^2 - 3k_3^3)x^3 + [(k_1^2 - k_2^2)^2 - 9k_1k_3^3]x^2 \\ - (k_1^2 - 2k_2^2)(k_1k_2^2 - 9k_3^3)x + (k_1^2 - 3k_2^2)(k_2^4 - 3k_1k_3^3).$$

I. SOLUTION BY W. D. CAIRNS, Oberlin College.

If  $k_1 = k_3 = 0$ , the expression can be factored thus:

$$(3x^4 + k_2^4)(x^2 - 3k_2^2).$$

If  $k_1 = k_2 = 0$ , it can be factored thus:

$$3x^3(x^3 + 2k_3^3) \text{ or } (3x^4 + 6k_3^3x)x^2.$$

If  $k_2 = k_3 = 0$ , it can be factored thus:

$$x^2(3x^2 - 3k_1x + k_1^2)(x^2 - k_1x + k_1^2).$$

These will be consistent only if one factor contains, at least, the terms  $3x^4 - 3k_1x^3 + k_1^2x^2 + 6k_3^3x + k_2^4$ , and the second  $x^2 - k_1x + k_1^2 - 3k_2^2$ . The last term of the given expression shows that the first factor must contain also the term  $-3k_1k_3^3$ , and multiplication of these two tentative factors shows that the term  $-k_1k_2^2x$  completes the first factor. Thus we have as the two factors:  $3x^4 - 3k_1x^3 + k_1^2x^2 - (k_1k_2^2 - 6k_3^3)x + (k_2^4 - 3k_1k_3^3)$  and  $x^2 - k_1x + (k_1^2 - 3k_2^2)$ .

II. REMARKS BY H. P. MANNING, Providence, R. I.

The polynomial can be written  $u_3k_3^3 + v_6$ , where  $u_3$  and  $v_6$  are polynomials of degrees 3 and 6 in  $x$ ,  $k_1$  and  $k_2$ . If there are rational factors  $k_3^3$  must go entirely with one factor, since  $u_3$  and  $v_6$  are not the cubes of rational expressions. The factor which does not contain  $k_3$  will be the highest common factor of  $u_3$  and  $v_6$ . We can also factor  $u_3$ , noting that it contains only the second power of  $k_2$ .

**2845 [1920, 326]. Proposed by E. L. POST, Princeton University.**

Prove that if  $y_x$  is a solution of the functional equation

$$y_x = \frac{y^2 x + 1}{x} + y_{x+1}$$

for positive integral values of  $x$  with  $y_x > 0$ , then

$$\lim_{x \rightarrow \infty} y_x \log x = 1.$$

SOLUTION BY THE PROPOSER, AND OTTO DUNKEL, Washington University.

If  $x$  and  $y_x$  are both positive, the equation

$$y_x = \frac{y^2 x + 1}{x} + y_{x+1} \tag{1}$$

has one positive root,

$$y_{x+1} = \frac{\sqrt{x^2 + 4xy_x} - x}{2}.$$

From (1), we clearly have then that

$$y_x > y_{x+1} > 0. \tag{2}$$

<sup>1</sup> Since the second difference  $y(a+2h) - 2y(a+h) + y(a) = h^2 y''(\theta)$ , where  $a < \theta < a+2h$ .—EDITORS.